$NN \to NN\pi$ reaction near threshold in a covariant one-boson-exchange model *

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Abstract

We calculate the cross sections for the $p(p, n\pi^+)p$ and $p(p, p\pi^0)p$ reactions for proton beam energies near threshold in a covariant one-boson-exchange model, which incorporates the exchange of π, ρ, σ and ω mesons and treats both nucleon and delta isobar as intermediate states. The final state interaction effects have also been included. The ω meson exchange is found to contribute significantly at these energies, which, along with other meson exchanges, provides an excellent agreement with the data. The cross sections at beam energies $\leq$ 300 MeV are almost free from the contributions of the Δ isobar excitation.

KEYWORD: π^0 and π^+ production near threshold, covariant one-boson-exchange model, contribution of heavy meson exchange.

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High precision data on the total as well as differential cross sections for the $p(p, p\pi^0)p$ and $p(p, n\pi^+)p$ reactions at the incident proton energies very close to the kinematical threshold have now become available [1,2,3]. At low energies, these reactions necessarily involve large momentum transfers, which make them very sensitive to the nucleon-nucleons (NN) interactions at small distances. These data provide a stringent test of the theories used to interpret pion production in elementary NN collisions.

Earlier attempts [4,5] to explain the π^0 data in the s-wave pion production approach of Koltun and Reitan [6], have not been successful as the calculations underpredicted the data by factors of 4-5. More recently, Lee and Riska [7] and Horowitz et al. [8] have shown that most of the theoretical underprediction found in earlier calculations, are removed if the exchange of heavy scalar and vector mesons is included in the calculations. However, both these sets of authors have difficulty (of varying degree) in explaining the energy dependence of the measured cross sections. An alternative explanation of the problem has been presented by Hernandez and Oset [9], who argue that if the off-shell properties of the pion-nucleon (πN) amplitude (which, in general, is much larger than its on-shell value) are taken into account explicitly while evaluating the pion- rescattering diagram, the calculated $p(p, p\pi^0)p$ cross sections come very close to the measured values. However, Hanhart et al. [10] show that with the off-shell πN amplitude calculated properly, the enhancement of the cross sections is much smaller than that reported in Ref. [9].

In the case of the $p(p, n\pi^+)p$ reaction, the models of Schillaci, Silbar and Young (SSY) [11], and Lee and Matsuyama (LM) [12] have been used to explain the data. SSY employ the partial conservation of axial vector current (PCAC) amplitude for the pion production with several approximations to reduce the complexity of the calculations. In the LM approach, the pion production via

the Δ isobar excitation is treated rather rigorously within a coupled channel formalism, while the nonresonant production process (which is important at low energies) is handled very approximately. Both these models are found to be inadequate to account for the data [3].

A proper theoretical understanding of the near threshold π^0 and π^+ production in *pp* collisons within one consistent picture is still lacking. The exchange of heavy scalar and vector mesons is important for the description of the short range part of the NN interaction [13], which should be included in the calculation of the pion production in NN collisions at low energies, in a fully covariant way. Most of the models mentioned above perform calculations in the non-relativistic framework where inaccuracies creep in also due to the ambiguity that exists in the non-relativistic reduction of the πNN Lagrangian [14].

In a recent publication [15], we have presented, in detail, a fully covariant effective one-boson-exchange model (CEOBEM) to describe the pion production in NN collisions. CEOBEM incorporates the exchange of π, ρ, σ and ω mesons and treats both nucleon and delta isobar excitations as intermediate states. The model involves only the physical parameters (like, coupling constants and cut-off masses), which are determined by fitting to the NN scattering data over a range of beam energies. Since we directly fit the NN scattering T-matrix, we have also included in CEOBEM a nucleon-nucleon-axial-vector-isovector meson vertex. This term provides an additional short range correlation which cures the unphysical behaviour in the angular distribution of NN scattering caused by the contact term in the one-pion exchange amplitude [15,16] in the limit of very large mass of the axial vector meson.

Within CEOBEM, it is possible to describe both the $p(p, p\pi^0)p$ and $p(p, n\pi^+)p$ reactions at beam energies near the threshold in one consistent picture, which is the aim of this letter. For applications at low energies, the model as presented in

Ref. [15], has to be extended to include the final state interaction (FSI) effects in the outgoing channel.

In the spirit of the Watson-Migdal theory of FSI [17], the transition amplitude for a $NN \to NN\pi$ type of reaction can be written as

$$A_{fi} \quad = \quad T_{fi}(NN \to NN\pi) T_{ff},$$

where $T_{fi}(NN \to NN\pi)$ is the primary production amplitude which describes the transition from initial NN state to the final $NN\pi$ state, while the amplitude T_{ff} accounts for the rescattering among the final particles. The essential approximation involved in this equation is the assumption that all the diagrams that lead to the pion production are contained in T_{fi}, while T_{ff} takes into account the elastic scattering among the final particles once they are produced by the processes involved in T_{fi}. Such a factorization of the transition amplitude, which can be justified under certain conditions by the general scattering theory [18], has been found to provide a good description of the near threshold $NN \to NN\eta$ reaction data [19].

We calculate the amplitude T_{fi} within the CEOBEM, summing coherently the contributions of various pion production diagramms in precisely the same way as discussed in Ref. [15], with all the parameters being the same as those given therein. On the other hand, the evaluation of the rescattering amplitude T_{ff} requires the solution of a set of integral equations involving two-body interactions among the different final state particles. However, we simplify the problem by identifying T_{ff} with the coherent sum of the two-body on-mass-shell elastic scattering amplitudes of the particles involved in the final channel [19]. Furthermore, we assume the pion-rescattering terms to be negligible. Indeed, several authors [7,8,10,20] agree with the fact that the on-shell part of this term is close to zero; the off-shell part of the πN rescattering is assumed to be contained in the πNN form factors [15]. Under these conditions, the T_{ff} is given by the inverse of the

Jost function, $J_\ell(k)$, for a given partial wave ℓ and the relative momentum k of the two outgoing nucleons [18,21].

The function $J_\ell(k)$ can be obtained by solving the Schrödinger equation with a given NN interaction with proper boundary conditions. As the FSI effects are most important at low relative energies, one can safely ignore the partial waves other that $\ell = 0$ [22] and (in the absence of Coulomb force) can make the following effective range expansion for the s-wave NN scattering phase-shift

$$k cot\delta_0 = \frac{1}{a_0} + \frac{1}{2} r_0 k^2,$$

where a_0 and r_0 are the scattering length and effective range parameters respectively. An analogous expression can be written also in the presence of the Coulomb interaction (see eg. Ref. [18]). Now, the inverse of the Jost function can be written as

$$(J_0(k))^{-1} = \frac{(k^2 + \alpha^2) r_0/2}{1/a_0 + (r_0/2)k^2 - ik},$$

where α is given by

$$\alpha = (1/r_0)[1 + (1 + 2r_0/a_0)^{1/2}].$$

The quantity $(J_0(k))^{-1}$ as written above, has the required property that it goes to unity for large k. This is, of course, to be expected since the final state interaction effects should disappear for large relative energies of the two outgoing nucleons. We have checked the accuracy of the Jost functions calculated in this way by comparing them with those obtained by solving the Schrödinger equation with the relevant parts of the Paris potential in some cases. There is a good general agreement between the two results. In case of the neutron-proton final channel, we have included both $^1S_0$ and $^3S_1$ FSI's, while for the proton-proton case only the former is considered. The corresponding scattering length and effective range parameters have been taken from Ref. [23].

In Fig. 1, we compare the results of our calculations (solid line) with the data for the total cross section of the $p(p, n\pi^+)p$ reaction as a function of the beam energy. We see that the agreement between calculations and the data is truly remarkable. Not only the absolute magnitude but also the energy dependence of the experimetal cross sections are well reproduced. Since our model includes the pion production via both the nucleon and Δ isobar intermediate states, it can be extended to higher beam energies in a straightforward way. We obtain a good agreement with the data also at higher energies ($E_{lab} > 320$ MeV). This is an unique feature of our calculations, as most of the recent models used to explain the near threshold pion production lack the treatment of both the non-resonant as well as resonant pion production consistently in a single framework. Therefore, the failure of these models to describe the data both at low and high beam energies is understandable.

In this figure, we also show the contributions of the various meson-exchange graphs to the total pion production cross sections. It is clear that the contribution of ω meson exchange is quite significant; very close to the threshold it is even larger than that of the pion exchange. In comparison to this, the strength of the scalar σ meson exchange is less, which is in contrast with the results of Horowitz et al. [8] who find the contribution of the σ meson exchange (calculated within the Bonn NN potential) to be larger. On the other hand, the results of Lee and Riska [7], with meson exchange terms calculated within the Paris potential, are in agreement with our findings. Of course, CEOBEM calculations do not involve any explicit dependence on the NN interaction model. Therefore, they can be used to distinguish between the predictions of the scalar and vector components of the heavy meson exchange contributions to the pion production calculated within the models dependent on NN interaction.

The comparison of our calculations with the data for the total cross sections

for $p(p, p\pi^0)p$ reaction is shown in Fig. 2. The solid curve represents the coherent sum of the considered diagramms corresponding to all the meson exchanges as described above (the relative contributions of the individual meson exchange graphs remain of the same kind as that in Fig. 1). The calculations are able to reproduce the absolute magnitude and beam energy dependence of the measured cross sections very well at the low as well as higher beam energies in this case too. Therefore, it is possible to have a consistent description of both π^+ and π^0 production in pp collisions within COEBEM, which has eluded most of the other theoretical models so far.

In addition, we show in this figure the relative contributions of the pion production via excitation of nucleon (dashed line) and Δ isobar (dotted line) intermediate states. Their coherent sum is shown by the solid line. We notice that non-resonant (nucleon intermediate state) pion production is predominant at beam energies near the threshold, while the resonant production becomes important as the beam energy increases. Thus the near threshold data taken at Bloomington and Uppsala are consistent with predominantly non-resonant pion production picture.

In Fig. 3, we compare our calculation with the data for the angular distribution of pions emitted in the reaction $p(p, n\pi^+)p$ at the beam energies of 320 MeV, 300 MeV and 294 MeV. The pion angles are in the πNN center-of-mass frame. The solid (dashed) lines show the calculated cross sections where contributions of both nucleon and delta isobar (only nucleon) intermediate states are included. There is a fairly good agreement between the theoretical and measured angular distributions. At beam energies of 300 MeV and 294 MeV, the contributions of the delta isobar excitation is negligible, while at 320 MeV delta isobar excitation makes a visible contribution to the cross section. The near isotropy of the calculated as well as measured cross sections at beam energies $\leq$ 300 MeV suggests

a s-wave pion final state at these energies. However, at 320 MeV there is some evidence of a non-s-wave contribution even in the non-resonant pion production. This could have consequences on the polarisation observables.

In summary, the covariant effective one-boson-exchange model with final state interactions among the outgoing nucleons taken into account, provides an excellent description of the recently measured data on π^+ and π^0 production in proton-proton collisions at beam energies near the kinematical threshold. The ω-meson exchange contributes significantly near the threshold. In CEOBEM, the heavy meson exchange contributions are independent of the NN interaction model. They can, thus, serve to distinguish between various predictions of the scalar and vector components of the short range axial-charge operator (which has the same form as that of the non-relativistic s-wave pion production) obtained with different NN interactions. This may have consequences in the explanation of the large enhancement of the effective axial charge, found in the analysis of first forbidden β transitions in heavy nuclei [24]. The data on the polarisation observables will be useful in underlining more details of the nature of the final state interactions and the neglected pion-rescattering term.

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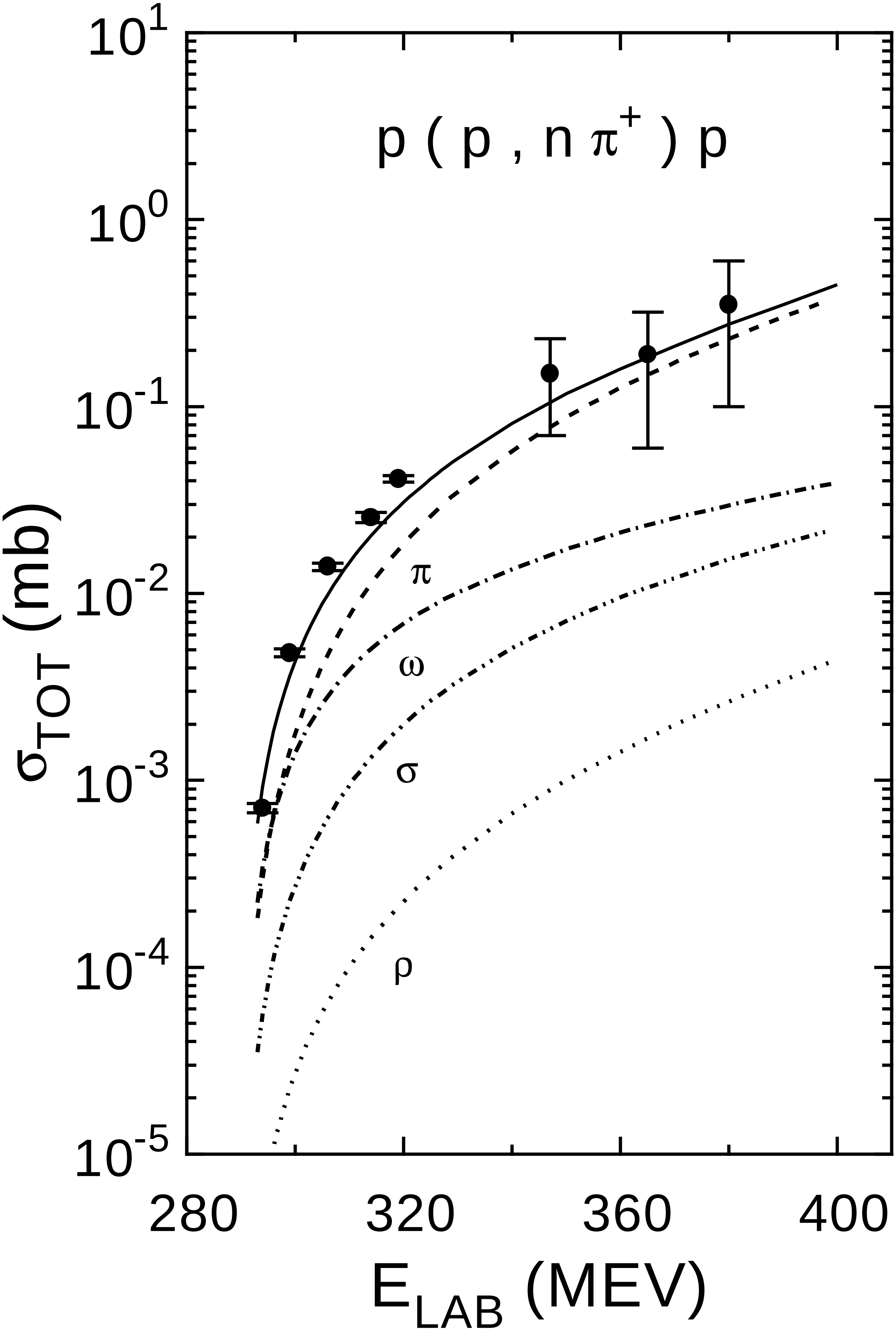
Figure Captions

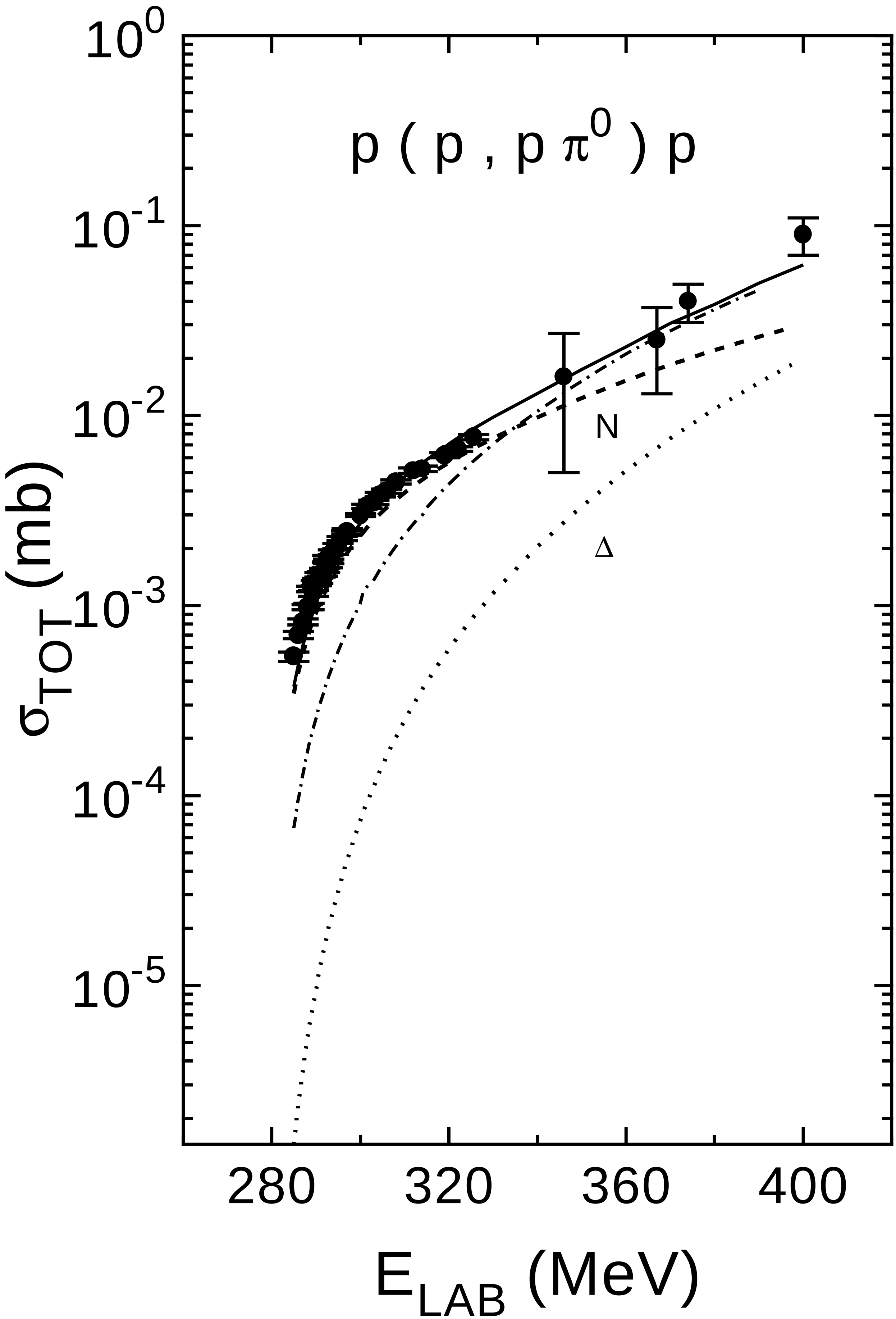
Fig. 1 The total cross section for the $p(p, n\pi^+)p$ reaction as a function of beam energy. The dotted, dashed- double-dotted, dashed-dotted and dashed curve represent the contributions of ρ, σ, ω and π meson exchanges respectively. Their coherent sum is shown by the solid line. The experimental data are from Ref. [3] and [25].

Fig. 2 The total cross section for the $p(p, p\pi^0)p$ reaction as a function of beam energy. The dotted and dashed curves represent the results of calculations obtained with delta only and nucleon only intermediate states respectively. Their coherent sum is shown by the solid line. The experimental data is from Ref. [1] and [25]. The dashed-dotted line shows the results of full calculations obtained with no final state interaction effects.

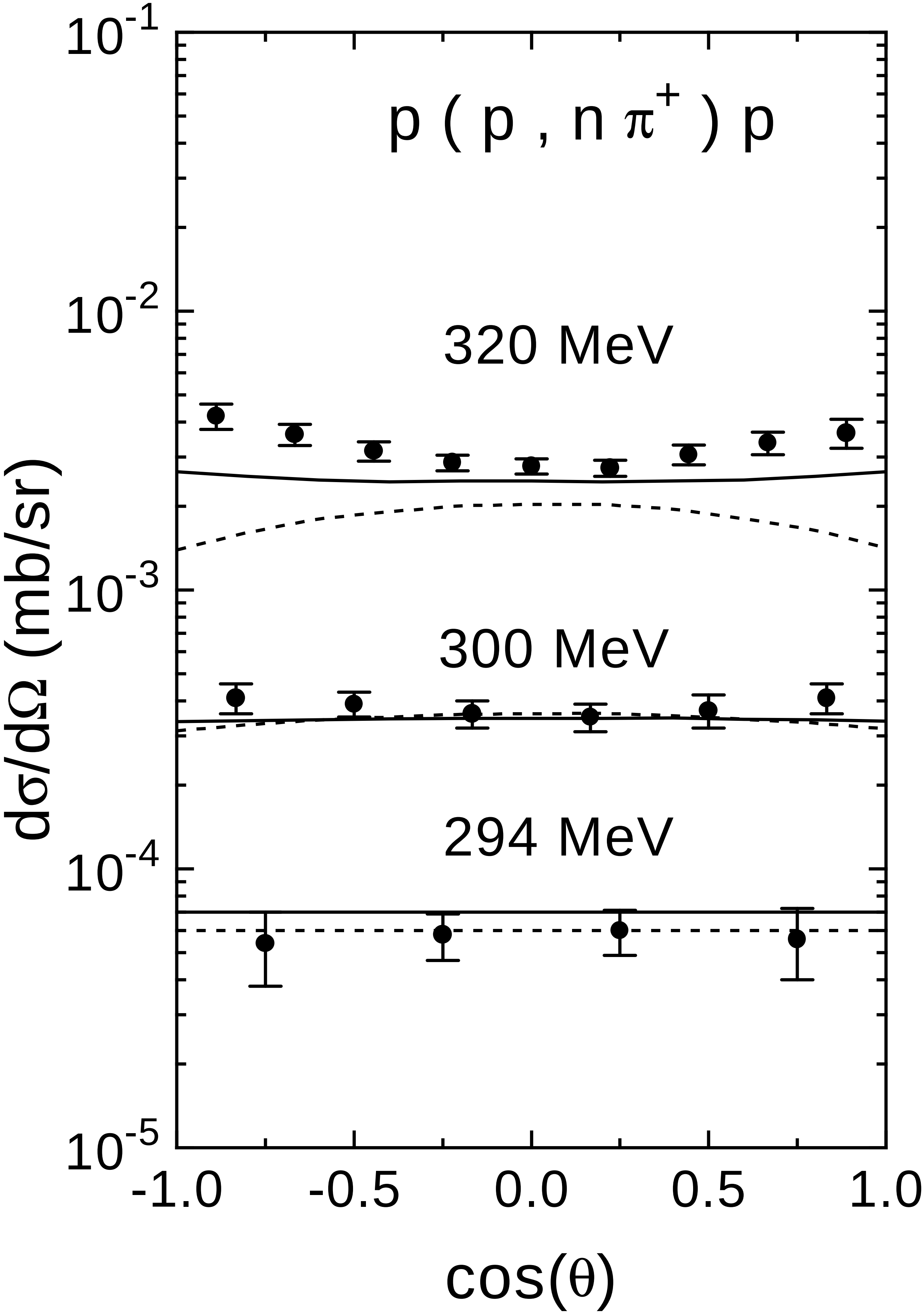
Fig. 3 Pion angular distributions in the πNN center-of-mass frame. The dashed lines represent the results of calculations obtained with nucleon only intermediate states while the solid line is the total cross sections in which the contributions of nucleon and delta intermediate states are coherently summed.



p (p , n π+) p
π
ω
σ
ρ
σTOT (mb)
ELAB (MEV)
10^1
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
280
320
360
400



10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
$p\,(p, p\,\pi^0)\,p$
N
Δ
σ_{TOT} (mb)
280
320
360
400
E_{LAB} (MeV)



p (p , n π+) p
320 MeV
300 MeV
294 MeV
dσ/dΩ (mb/sr)
10-1
10-2
10-3
10-4
10-5
-1.0
-0.5
0.0
0.5
1.0
cos(θ)